

A COMPARATIVE STUDY FOR MULTI-PERIOD ASSET ALLOCATION OF DEFINED CONTRIBUTION SCHEMES: EVIDENCE FROM TURKEY

Kerem Şenel¹

A. Bülent Pamukçu²

ABSTRACT

Long term asset allocation is more complicated than the usual asset allocation paradigm due to the multi-period nature of the problem. Since analytical models usually oversimplify for the sake of mathematical convenience, a fundamentally different approach is needed. Numerical solutions such as genetic algorithms provide an important alternative to analytical solutions in handling the inherent complexity which manifests itself as discontinuities and nonlinearities of the solution space. Complementing our previous studies with hypothetical and US market data, this study brings further evidence on the comparative performance of numerical solutions for multi-period asset allocation from an emerging market.

Keywords: Asset allocation, pension fund, life cycle fund, genetic algorithm

BİREYSEL EMEKLİLİK FONLARINDA ÇOK DÖNEMLİ VARLIK TAHSİSİ İÇİN TÜRKİYE ÜZERİNE KARŞILAŞTIRMALI BİR ÇALIŞMA

ÖZET

Uzun dönemli varlık tahsisi, çok dönemli doğası nedeni ile, klasik varlık tahsisi paradigmasından çok daha karmaşıktır. Analitik modeller sorunu matematiksel gerekçelerle aşırı derecede basitleştirme yoluna gittiğinden temelde çok daha farklı bir yaklaşıma ihtiyaç duyulmaktadır. Kendini doğrusal olmayan bir çözüm uzayında süreksizliklerle gösteren bu karmaşıklığı çözebilmek için genelde sayısal yöntemler, özel olarak da genetik algoritmalar önemli bir alternatif olarak karşımıza çıkmaktadır. Daha önce hipotetik ve ABD verileri ile yaptığımız çalışmalarını tamamlayıcı bir niteliğe sahip olan bu çalışma ile, sayısal yöntemlerin çok dönemli varlık tahsisi sorununu çözme performansı hakkında daha fazla kanıt elde etme amacını güdüyoruz.

Anahtar Kelimeler: Varlık tahsisi, bireysel emeklilik fonu, yaşam döngüsü fonu, genetik algoritma

¹ Adjunct Professor, Boğaziçi University, İstanbul, Turkey

² Professor, İstanbul Commerce University, İstanbul, Turkey

1. INTRODUCTION

Defined contribution pension plans are fast becoming the norm rather than exception. Szomlyan (2008) shows that, by the year 2015, the DC market in US will nearly double in size to reach \$7.8 trillion in assets under management. The trend is not confined to developed markets only. For instance, in Turkey, since all employees are obliged by law to participate in the government-run social security system, defined contribution pension plans are supplementary. Yet, despite its supplementary status, the number of participants in defined contribution pension plans has drastically increased from 0.3 million in 2004 to 2.7 million in 2012 according to data from Emeklilik Gözetim Merkezi (Pension Monitoring Center; an institution which renders data regarding the pension fund industry in Turkey).

The increasing popularity of defined contribution is mainly due to its flexible nature which allows faster vesting in the event of a job change and access to funds in the event of a job loss or other financial hardship. The price of this flexibility is the shift of asset allocation responsibility to individuals. Individuals typically follow a naive strategy by dividing their contributions evenly across the funds offered in the plan (Benartzi and Thaler, 2001). Hence, the proportion invested in stocks turns out to be high if there are a large number of stock funds available to the investor, or vice versa. However, asset allocation in the context of defined contribution pension schemes requires a highly sophisticated approach. The multi-period nature of long term asset allocation is where complication arises.

As demonstrated by Sherris (1993), it is evident that a pension portfolio will need rebalancing as time passes and actual asset returns are revealed. If realized returns turn out to be higher than expected returns and the current balance of funds points to an ultimate surplus, then, high risk assets can be avoided and the portfolio can predominantly be composed of low risk assets. On the other hand, if realized returns turn out to be low and the current balance of the fund suggests an ultimate deficit, then, the situation may call for higher portfolio weights for high risk assets in order to catch up with the long term target.

Besides rebalancing according to past performance, individuals are naturally expected to gradually shift their funds toward less risky assets as they approach retirement. In reality, many participants never reallocate their investments or even review their allocation. A solution for this problem has been the invention of life cycle funds. These funds set a target maturity date and gradually shift the investments from aggressive to conservative. Typically, a fund will start with a heavy allocation to high risk assets, i.e. equities, and, over time, the allocation will move from primarily high risk assets to a balanced portfolio of high risk and low risk assets and to primarily low risk assets, i.e. fixed income.

In practice, life cycle allocation and threshold (funded status) allocation are two widely used strategies that are employed for the asset allocation of life cycle funds. Blake

et al. (1999) explains these strategies in detail. In a typical life cycle allocation, funds are 100% invested in high risk assets for, say, the first 20 years. Then, this is followed by a 5% per year switch into low risk assets during the last 20 years. Although this is a dynamic strategy, it does not include a feedback mechanism for rebalancing. The threshold allocation strategy, on the other hand, incorporates a simple form of feedback control. In this strategy, 100% of the funds are invested in high risk assets if the size of the fund is below a lower threshold so that the fund hopefully benefits from higher expected returns. If the size of the fund is above a higher threshold, then 100% of the funds are invested in low risk assets to safeguard the current level of funds. In between, i.e. when the size of the fund is between the lower and higher thresholds, the proportion of funds invested in low risk assets increases linearly as the size of the fund increases.

This paper is organized as follows. First, we review the literature. Second we introduce a model for the defined contribution pension scheme that we use in our simulations. The next section focuses on the genetic algorithm and alternative solutions. Then, we present the experiment setup and the simulation results. Finally, we present our conclusions.

2. LITERATURE REVIEW

2.1. Classical Contributions to Asset Pricing and Allocation

Earlier research generally focuses on the impact of the choice of utility function on asset allocation.

Markowitz (1952, 1987) introduces the one-period mean-variance principle to create a portfolio of investments with a set of optimal risk-return combinations to maximize the expected utility of ending wealth for risk-averse individuals. This method yields inefficient results when the investment horizon is greater than one period.

Mossin (1968) indicates that asset allocation with more than one period is optimal for a logarithmic utility function, but suboptimal for other utility functions.

Merton (1969) studies the combined problem of optimal portfolio selection and consumption rules for an individual in a continuous-time model by examining a two-asset model with constant relative risk aversion. In this model, the individual's income is generated by capital gains on investments in assets with prices assumed to satisfy the geometric Brownian motion hypothesis.

Merton (1971) extends the results from Merton (1969) for more general utility functions, price behavior assumptions, and for income generated also from non-capital gains sources.

Merton (1973) develops an intertemporal capital asset pricing model from the portfo-

lio selection behavior by an arbitrary number of investors who act so as to maximize the expected utility of lifetime consumption and who can trade continuously in time. It is shown that, unlike the one-period model, current demands are affected by possibility of uncertain changes in future investment opportunities. It is also shown that, contrary to the classical capital asset pricing model, expected returns on risky assets may differ from the riskless rate even when they have no systematic or market risk.

Merton (1992) recaps previous work and introduces a number of further developments such as the Cox-Huang methodology for solving the optimal consumption-portfolio problem that does not require differentiability of the derived-utility function.

2.2 Analytical Multi-Period Asset Allocation

Contemporary research using analytical models generally focuses on risk measures for measuring portfolio performance, particularly in defined contribution and defined benefit pension schemes.

Vigna and Haberman (2001) and Haberman and Vigna (2002) develop analytical solutions to the multi-period asset allocation problem in the context of defined contribution pension schemes by using dynamic programming. Confirming the current industry practice, simulations of analytical models suggest a gradual switch from high risk to low risk assets as the individual approaches retirement. Unfortunately, these analytical solutions typically fail to incorporate real world constraints such as short selling restrictions for the sake of mathematical convenience.

Tuncer and Senel (2004) provide a solution to the multi-period asset allocation problem in the context of defined contribution pension schemes without recourse to dynamic programming. Their solution also takes GARCH effects into account and, hence, generalizes to time varying volatility. They show that an analytical model that fails to incorporate GARCH effects leads to suboptimal portfolio performance.

Hibiki (2006) compares the two types of multi-period stochastic optimization models, i.e. the hybrid model and the scenario tree model, and clarifies that the hybrid model can evaluate and control risk better than the scenario tree model using some numerical tests. According to the numerical results, an efficient frontier of the hybrid model with the fixed-proportion strategy dominates that of the scenario tree model when they are evaluated on simulated paths.

Kung (2008) uses stochastic dynamic programming to set up a multi-period asset allocation model and derives an analytic formula for the optimal proportions invested in short and long bonds. It is shown that an investor will hold a larger proportion of short bond if his/her investment horizon gets shorter and/or if he/she is more risk averse.

Huang and Lee (2010) propose an optimization approach to generate the optimal investment strategy of an asset liability management model for long-term endowment

policies. The proposed discrete time investment model includes both static, single-period and multi-period optimal investment strategies for a time-dependent asset return process. In the case of the multi-period method, a first approximation of the optimal asset allocation is obtained, as applied to a ceased life insurance product line. The numerical results show that the proportion of cash should increase when a portfolio with uniform years before maturity is compared with a portfolio comprised of new policies.

Josa-Fombellida and Rincon-Zapatero (2010) study the optimal management of an aggregated pension fund of defined benefit type, in the presence of a stochastic interest rate. The results obtained are based on the analytical solutions found by means of the dynamic programming approach. A numerical simulation of the model shows that the debt is reduced by means of risky investment in the first years and with a more conservative investment policy in the last years of the planned period.

2.3 Stochastic Multi-Period Asset Allocation Research with Evolutionary Algorithm

Complications due to real life constraints or market imperfections are almost always neglected by analytical models for the sake of mathematical tractability. These have been addressed by a number of studies that utilize evolutionary algorithm and other stochastic search techniques.

Baglioni et al. (2000) uses an evolutionary algorithm for multi-period asset allocation in order to handle the complications due to regulatory constraints, individual investor's requirements, non-trivial indices of risk, and subjective quality measures.

Senel et al. (2006, 2008) advocate the use of genetic algorithms and stochastic search techniques to better handle the inherent complexity of the problem which manifests itself as discontinuities and nonlinearities of the solution space. Discontinuities arise due to short selling restrictions or other potential constraints such as upper limits for individual assets or asset classes. Nonlinearities are due to quadratic terms in portfolio variance for classical portfolio optimization or nonlinear cost (disutility) functions, which are commonly used for pension portfolio optimization. Senel et al. (2006, 2008) show that genetic algorithms and stochastic search techniques outperform analytical models as well as heuristic methods such as life cycle allocation and threshold (funded status) allocation in terms of a number of risk measures such as probability of failure, fifth percentile (value at risk), and mean shortfall (conditional value at risk).

Zhang and Zhang (2009) claim to improve Hibiki's model by introducing the risk measure CVaR to control the wealth loss risk while maximizing the expected utility, by considering typical market imperfections such as short sale constraints and proportional transaction costs simultaneously, and by applying a genetic algorithm to solve the resulting model.

3. THE MODEL

In our defined contribution pension scheme, funds can be invested in n assets with different levels of risk and expected return. Contributions, which are assumed to be a fixed percentage of income, are paid at the beginning of each period. Retirement is assumed to be the only decrement from accumulated funds. Taxation is not taken into account; i.e., contributions and investment income are assumed to be tax-exempt.

We assume that the final fund (the level of accumulated funds at retirement), f_{final} , is converted into a whole life annuity due. Using the retiree's expected mortality and the return of the low-risk asset (fixed-income) as the discount rate, the annual retirement income is given by

$$\text{Annual Retirement Income} = \frac{f_{\text{final}}}{\ddot{a}_x} \tag{1}$$

where \ddot{a}_x is the actuarial present value of a whole life annuity of 1 payable at the beginning of each year (starting immediately after retirement) as long as the retiree who is at the age of x at retirement survives. The actuarial present value of a whole life annuity of 1 payable at the beginning of each year, \ddot{a}_x , is given by

$$\ddot{a}_x = \sum_{k=0}^{\omega} {}_k p_x v^k \tag{2}$$

where $\omega + x$ is the maximum age, ${}_k p_x$ is the probability that a retiree who is at the age of x (at retirement) survives for k years, and v is the discount factor that uses the return of the low-risk asset (fixed income). The discount factor, v , is given by

$$v = E(e^{-X}), X \sim N(\mu_{\text{low-risk asset}}, \sigma_{\text{low-risk asset}}^2) \tag{3}$$

Since the final fund is assumed to be converted into an annuity, the retiree will be concerned with the net replacement ratio, which is defined as the ratio of retirement income to final salary. Assuming that the real salary level is set equal to 1 and the real salary growth rate is zero, the net replacement ratio is given by

$$\text{Net Replacement Ratio} = \frac{\text{Retirement Income}}{\text{Final Salary}} = \frac{\frac{f_{\text{final}}}{\ddot{a}_x}}{1} = \frac{f_{\text{final}}}{\ddot{a}_x} \tag{4}$$

4. THE GENETIC ALGORITHM AND ALTERNATIVE SOLUTIONS

4.1. The Genetic Algorithm

A genetic algorithm is a search technique inspired by biological evolution. Genetic algorithms (or sometimes called evolutionary algorithms) were introduced by Holland (1975) as a method to perform randomized global search in a solution space. As McNelis (2005) indicates, the usefulness of classical optimization techniques such as Newton-based optimization (including backpropagation) and simulated annealing crucially depend on how good the initial parameter guess is. Hence, these techniques are vulnerable to the possibility of getting trapped at local minima. On the other hand, the likelihood of landing in a local minimum is greatly reduced with a genetic algorithm. The genetic algorithm does not require the approximation of Hessian matrices as in the case of Newton-based optimization. Hence, it is a statistical search process like simulated annealing. Yet, the term statistical does not suffice to describe the genetic algorithm, since the process is also evolutionary. Hence, the genetic algorithm is better termed as evolutionary stochastic search.

A typical genetic algorithm emulates evolutionary mechanisms such as “selection”, “crossover”, “mutation”, and “survival of the fittest”. Potential solutions, which comprise the “population”, evolve into the next generation of solutions by an evaluation of how fit they are via a fitness function.

The genetic algorithm and alternative solutions have been implemented in R 2.5.1 (2007-06-27). R includes an “R Based Genetic Algorithm” for binary and floating point chromosomes. We used the floating point chromosome version of the algorithm to minimize the expected cost (disutility) function for the next period.

The cost function is defined in Vigna and Haberman (2001), Haberman and Vigna (2002), and Tuncer and Senel (2004) as

$$C_t = (F_t - f_t)^2 + \alpha(F_t - f_t) \quad (5)$$

C_t : Cost incurred at the end of period t

F_t : Target level for accumulated funds at the end of period t

f_t : Actual level of accumulated funds at the end of period t

α : Risk aversion parameter

where $\alpha \geq 0$.

In short, negative deviations from the target level are always penalized whereas positive deviations from the target level may be rewarded or penalized depending on the size of the deviation. Large positive deviations are penalized in order to prevent excessive risk exposure. Risk aversion decreases with increasing risk aversion parameter α . In the limit when α approaches infinity, even large positive deviations are not penalized. The target levels are given a priori.

The optimization problem can be stated as

$$\min E[C_{t+1}|I_t] \tag{6}$$

I_t : Information available at time t

The arguments for the “R Based Genetic Algorithm” are as follows:

stringMin: (vector with minimum values for each gene) Each gene corresponds to the weight of an asset in the portfolio. Since short selling is not allowed, we have set stringMin to a vector of 0's.

stringMax: (vector with maximum values for each gene) We have set stringMax to a vector of 1's.

popSize: (population size) The population size is set equal to 50.

iters: (number of iterations) The number of iterations is set equal to 200.

In order to ensure that the sum of portfolio weights adds up to 1, the cost function uses a transformed vector which is simply the original vector used by the genetic algorithm divided by the sum of its elements.

For the following arguments, we have used the default values:

mutationChance: (chance that a gene in the chromosome mutates) The default value is $1/(\text{length of the chromosome}+1)$.

elitism: (number of chromosomes that are kept into the next generation) The default value is 20% of the population size.

First of all, an initial population is randomly generated by using uniform distribution for obtaining values between the limits imposed by stringMin and stringMax. Then, a fitness score is produced for each member of the population. Since elitism is employed, the fittest members of the population are saved for the next generation. The other members of the next generation are obtained by single-point crossover and a nonuniform mutation operator with a decreasing dampening factor over time, which provides an opportunity for fine tuning as the potential solutions approach a global optimum.

4.2. The Analytical Model

We use an analytical model which minimizes the same expected cost (disutility) function as the genetic algorithm. The analytical model is developed in Tuncer and Senel (2004) and used with genetic algorithms and other stochastic search techniques in Senel et al. (2006, 2008).

As mentioned before, the problem with analytical solutions is that they typically fail to incorporate real world constraints such as short selling restrictions for the sake of mathematical convenience.

Since the analytical model we use does not take into account the short selling restriction, the portfolio weights should be truncated to $[0, 1]$. The algorithm devised for this truncation procedure is described in Senel et al. (2006, 2008).

4.3. The Simulated Annealing Algorithm

The simulated annealing algorithm was initially developed by Metropolis et al. (1953) as a modified Monte Carlo scheme for multidimensional integrals in statistical physics. Then, it was generalized by Kirkpatrick et al. (1983) to solve nonlinear problems. Then, as indicated by Ingber (1993), it evolved into a popular stochastic search technique that is especially useful for dealing with cost functions possessing quite arbitrary degrees of nonlinearities, discontinuities, and stochasticity.

The name “simulated annealing” is due to the analogy between annealing, i.e. the cooling of a metal into a minimum energy structure, and the way the simulated annealing algorithm converges to the result. Ingber indicates that the algorithm we use is known as Boltzmann annealing as it utilizes a probability function similar to the probability of increase in energy due to the law of thermodynamics. For the algorithmic flow, see Senel et al. (2008).

4.4. The Life Cycle Allocation Strategy

Blake et al. (1999) provide an example of a typical life cycle allocation, in which funds are 100% invested in high risk assets for, say, the first 20 years. Then, this is followed by a 5% per year switch into low risk assets during the last 20 years. The portfolios of high and low risk assets are constructed by regular Markowitz portfolio optimization, where the target return for each portfolio is set at a rate that is equal to the mean of the arithmetic asset returns of the portfolio constituents. Although this is a dynamic strategy, it does not include a feedback mechanism for rebalancing.

4.5. The Threshold (Funded Status) Strategy

Blake et al. (1999) describe threshold allocation, on the other hand, as a strategy that

incorporates a simple form of feedback control. In this strategy, 100% of the funds are invested in high risk assets if the size of the fund is below a lower threshold so that the fund hopefully benefits from higher expected returns. If the size of the fund is above a higher threshold, then 100% of the funds are invested in low risk assets to safeguard the current level of funds. In between, i.e. when the size of the fund is between the lower and higher thresholds, the proportion of funds invested in low risk assets increases linearly as the size of the fund increases. The portfolios of high and low risk assets are constructed in exactly the same way as for the life cycle allocation strategy. We set the lower and upper threshold levels at 0.40 and 0.80, respectively.

5. EXPERIMENT SETUP

5.1. Asset Selection and Data

We employ two different setups to carry out our simulations. In the first setup, we use three indices which represent the major asset classes relevant for Turkish investors. These are the Bloomberg 5-7 years US bond index, the KYD 365 days Turkish bond index, and the Ulusal 30 Turkish stock index. In the second setup, the stock index is substituted by eight different stocks which are randomly selected from the constituents of the stock index. The ticker codes of these stocks are ARCLK, ISCTR, ISGYO, KRDM, MIGRS, SISE, GARAN, and HURGZ. The second setup is used for testing the comparative performances of alternative strategies as the problem becomes more complicated with increasing number of assets. We use daily price data from the period June 30, 2003 to July 24, 2008.

5.2. Simulations

We set the initial fund level to zero and the contribution rate to 8% of the salary. We make our simulations for a total of 20 years. The individual is assumed to retire at the age of 65 and the 1983 US GATT (unisex) mortality table is used for calculating the individual's expected mortality. We set the annual target rate of return for calculating interim fund level targets to the Chisini average of asset returns. The Chisini average of gross asset returns is given by

$$E \left[e^{\left(\frac{1}{n} \right) \sum_{i=1}^n X_{it}} \right], X_{it} \sim N(\mu_i, \sigma_i^2), i = 1, 2, \dots, n \tag{7}$$

where n is the number of available assets. Short selling is not allowed; i.e., portfolio weights are limited within $[0, 1]$. The risk aversion parameter α in the cost function is set equal to 10, 25, and 50 for high, medium, and low risk aversion levels, respectively. For two different setups with three and ten assets and three different risk aversion levels, there are six different cases under consideration. By using the genetic algorithm

and alternative methods, 100 simulations for each case, i.e. a total of 600 simulations, are carried out to simulate the portfolio weights, the interim and final fund levels, and the net replacement ratio.

Then, the comparative performances of alternative strategies are evaluated by using a number of risk measures, i.e. probability of failure, fifth percentile (value at risk), and mean shortfall (conditional value at risk), which are explained in Haberman and Vigna (2002), Tuncer and Senel (2004), and Senel et al. (2006, 2008).

6. SIMULATION RESULTS

6.1. Distributional Properties

Table 6.1.1 summarizes the distributional properties of the simulated net replacement ratios for the first setup with three assets.

Table 6.1.1 Distributional Properties of NRRs (Three Assets, High, Medium, and Low Risk Aversion, Target NRR = 0.45)

High Risk Aversion	Mean	Std. Dev.	Min.	Max.
Analytical	0.80	0.09	0.59	1.12
Genetic Algorithm	0.78	0.22	0.12	1.11
Simulated Annealing	0.78	0.22	0.12	1.11
Life Cycle	1.03	1.48	0.13	14.20
Threshold	0.89	0.41	0.12	2.04
Medium Risk Aversion	Mean	Std. Dev.	Min.	Max.
Analytical	0.88	0.14	0.55	1.34
Genetic Algorithm	1.11	0.49	0.10	1.94
Simulated Annealing	1.11	0.49	0.10	1.94
Life Cycle	0.98	0.92	0.12	4.91
Threshold	0.87	0.41	0.10	2.09
Low Risk Aversion	Mean	Std. Dev.	Min.	Max.
Analytical	0.87	0.14	0.59	1.29
Genetic Algorithm	1.56	0.90	0.21	3.04
Simulated Annealing	1.57	0.90	0.21	3.04
Life Cycle	0.95	0.94	0.15	5.41
Threshold	0.89	0.38	0.21	2.27

For the three asset case, although the analytical model and the threshold strategy have similar mean net replacement ratios, the analytical model has a very low standard deviation compared to the other strategies.

On the other hand, the genetic algorithm and the simulated annealing method generate similar distributional properties for the net replacement ratio. For medium and low risk aversion, these strategies have the highest mean net replacement ratio.

For high risk aversion, although the lifecycle strategy has the highest mean net replacement ratio, it also has the highest standard deviation.

Table 6.1.2 summarizes the distributional properties of the simulated net replacement ratios for the second setup with ten assets.

Table 6.1.2 Distributional Properties of NRRs (Ten Assets, High, Medium, and Low Risk Aversion, Target NRR = 2.96)

High Risk Aversion	Mean	Std. Dev.	Min.	Max.
Analytical	0.86	0.13	0.60	1.15
Genetic Algorithm	3.02	0.46	1.77	5.02
Simulated Annealing	3.00	0.39	2.00	4.52
Life Cycle	2.04	1.82	0.26	8.17
Threshold	1.14	0.31	0.42	1.99
Medium Risk Aversion	Mean	Std. Dev.	Min.	Max.
Analytical	0.85	0.12	0.61	1.15
Genetic Algorithm	3.75	0.28	3.02	4.52
Simulated Annealing	3.79	0.25	3.09	4.31
Life Cycle	2.71	8.06	0.15	76.53
Threshold	1.08	0.33	0.21	2.22
Low Risk Aversion	Mean	Std. Dev.	Min.	Max.
Analytical	0.85	0.14	0.47	1.28
Genetic Algorithm	4.84	0.41	4.11	7.14
Simulated Annealing	4.93	0.29	4.35	6.15
Life Cycle	1.79	1.59	0.26	9.75
Threshold	1.14	0.38	0.38	2.65

For the ten asset case, the analytical model has the lowest mean net replacement ratio with the lowest standard deviation among all the strategies.

The genetic algorithm and the simulated annealing method generate similar distributional properties for the net replacement ratio. At all risk aversion levels, these strategies have the highest mean net replacement ratio.

The lifecycle strategy has the highest standard deviation.

For both the three asset and ten asset cases, lower risk aversion increases the mean net replacement ratio for the genetic algorithm and the simulated annealing model.

6.2. Risk Measures

Table 6.2.1 summarizes the risk measures computed for the simulated net replacement ratios for the first setup with three assets.

Table 6.2.1 Risk Measures for NRRs (Three Assets, High, Medium, and Low Risk Aversion, Target NRR = 0.45)

High Risk Aversion	Probability of Failure	Mean Shortfall	5 th Percentile
Analytical	0%	NaN	0.63
Genetic Algorithm	11%	0.20	0.22
Simulated Annealing	11%	0.20	0.22
Life Cycle	31%	0.16	0.17
Threshold	18%	0.20	0.16
Medium Risk Aversion	Probability of Failure	Mean Shortfall	5 th Percentile
Analytical	0%	NaN	0.68
Genetic Algorithm	17%	0.19	0.20
Simulated Annealing	17%	0.19	0.20
Life Cycle	31%	0.15	0.21
Threshold	19%	0.17	0.20
Low Risk Aversion	Probability of Failure	Mean Shortfall	5 th Percentile
Analytical	0%	NaN	0.67
Genetic Algorithm	18%	0.14	0.27
Simulated Annealing	18%	0.14	0.27
Life Cycle	33%	0.13	0.22
Threshold	17%	0.14	0.28

For the three asset case, the analytical model has the best risk measures. The probability of failure is virtually zero (no failure in 300 simulations). Also, the analytical model has the highest fifth percentile.

Risk measures for the genetic algorithm, the simulated annealing method, and the threshold strategy are similar.

The life cycle strategy has the highest probability of failure among all the strategies.

Table 6.2.2 summarizes the risk measures computed for the simulated net replacement ratios for the second setup with ten assets.

Table 6.2.2 Risk Measures for NRRs (Ten Assets, High, Medium, and Low Risk Aversion, Target NRR = 2.96)

High Risk Aversion	Probability of Failure	Mean Shortfall	5 th Percentile
Analytical	100%	2.10	0.65
Genetic Algorithm	41%	0.34	2.34
Simulated Annealing	47%	0.26	2.31
Life Cycle	78%	1.70	0.38
Threshold	100%	1.82	0.61
Medium Risk Aversion	Probability of Failure	Mean Shortfall	5 th Percentile
Analytical	100%	2.11	0.66
Genetic Algorithm	0%	NaN	3.19

Simulated Annealing	0%	NaN	3.35
Life Cycle	85%	1.82	0.31
Threshold	100%	1.88	0.39
Low Risk Aversion	Probability of Failure	Mean Shortfall	5th Percentile
Analytical	100%	2.10	0.66
Genetic Algorithm	0%	NaN	4.27
Simulated Annealing	0%	NaN	4.52
Life Cycle	88%	1.63	0.44
Threshold	100%	1.81	0.64

For the ten asset case, the probability of failure for the analytical model and the threshold strategy is 100% (300 failures in 300 simulations).

On the other hand, risk measures for the genetic algorithm and the simulated annealing method are very similar. The probability of failure for these strategies is substantially lower than the probability of failure for the other strategies (no failure in 200 simulations for the medium and low risk aversion levels). Also, these strategies have the highest fifth percentile.

For the three asset case, risk aversion level has no significant impact on risk measures. For the ten asset case, lower risk aversion decreases the probability of failure and increases the fifth percentile for the genetic algorithm and the simulated annealing method.

7. CONCLUSIONS

The simulation results show that one model does not fit all. Although the analytical model works fine for a small number of assets, it is also the worst alternative for a large number of assets. Since the analytical model does not incorporate short selling restriction and other restrictions for the sake of mathematical convenience, it yields suboptimal results as the problem gets more complicated.

The genetic algorithm and other similar methods work comparatively best for non-linear problems with discontinuous search spaces, such as the problem of long-term asset allocation in pension funds with a large number of assets and numerous potential restrictions.

It is best to decide which strategy to employ by using the help of simulation results. Other than the number of assets, there may be a lot of potential factors that affect the relative performance of different strategies. Such potential factors include the correlation structure, length of contribution period, and upper and lower limits on portfolio weights.

Our results with Turkish data confirm the results from our previous studies with hypothetical and US data. The results show a significant potential for the use of genetic algorithms in multi-period asset allocation.

REFERENCES

- Baglioni S., Da Costa Pereira C., Sorbello D., Tettamanzi A. G. B., (2000), "An Evolutionary Approach to Multiperiod Asset Allocation", *Lecture Notes in Computer Science*, (1802), 225-236.
- Benartzi S., Thaler R. H., (2001), "Naive Diversification Strategies in Defined Contribution Saving Plans", *American Economic Review*, 91(1), 79-98.
- Blake D., Cairns A., Dowd K., (1999), "Pension Metrics: Stochastic Pension Plan Design and Value at Risk During the Accumulation Phase", BSI-Gamma Foundation (Global Asset Management Methods And Applications), Working Paper Series, (11), 1-77.
- Haberman S., Vigna E., (2002), "Optimal Investment Strategies and Risk Measures in Defined Contribution Pension Schemes", *Insurance: Mathematics and Economics*, (31), 35-69.
- Hibiki N., (2006), "Multi-period stochastic optimization models for dynamic asset allocation", *Journal of Banking & Finance*, (30), 365-390.
- Holland J. H., (1975), **Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial intelligence**, Ann Arbor, MI, Michigan University Press.
- <http://www.egm.org.tr/weblink/BESgostergeler.htm>, [2 February 2012, WEB].
- Huang H., Lee Y., (2010), "Optimal asset allocation for a general portfolio of life insurance policies", *Insurance: Mathematics and Economics*, (46), 271-280.
- Ingber L., (1993), "Simulated Annealing: Practice versus Theory", *Mathematical and Computer Modeling*, 18(11), 29-57.
- Josa-Fombellida R., Rincon-Zapatero J. P., (2010), "Optimal asset allocation for aggregated defined benefit pension funds with stochastic interest rates", *European Journal of Operational Research*, (201), 211-221.
- Kirkpatrick S., Gelatt C. D. Jr., Vecchi M. P., (1983), "Optimization by Simulated Annealing", *Science*, 220(4598), 671-680.
- Kung J. J., (2008), "Multi-period asset allocation by stochastic dynamic programming", *Applied Mathematics and Computation*, (199), 341-348.
- Markowitz H. M., (1987), **Mean-Variance Analysis in Portfolio Choice and Capital Markets**, Oxford, UK, Basil Blackwell.
- Markowitz H. M., (1952), "Portfolio selection", *Journal of Finance*, (7), 77-91.
- McNelis P. D., (2005), **Neural Networks in Finance: Gaining Predictive Edge in the Market**, Burlington, MA, Elsevier.
- Merton R. C., (1969), "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case", *The Review of Economics and Statistics*, (51), 247-257.
- Merton R. C., (1971), "Optimum consumption and portfolio rules in a continuous-time model", *Journal of Economic Theory*, (3), 373-413.
- Merton R. C., (1992), **Continuous-Time Finance**, Revised Edition, Malden, MA, Blackwell Publishing.
- Merton R. C., (1973), "An intertemporal capital asset pricing model", *Econometrica*, (41), 867-887.
- Metropolis N., Rosenbluth A. W., Rosenbluth M. N., Teller A. H., Teller E., (1953), "Equation of State Cal-

culations by Fast Computing Machines”, *Journal of Chemical Physics*, 21(6), 1087-1092.

Mossin J., (1968), “Optimal multiperiod portfolio policies”, *Journal of Business*, (41), 215–229.

R Development Core Team, (2007), **R: A language and environment for statistical computing**, Vienna, Austria, R Foundation for Statistical Computing.

Senel K., Pamukcu A. B., Yanik S., (2006), “Using Genetic Algorithms for Optimal Investment Allocation Decision in Defined Contribution Pension Schemes”, 10th International Congress on Insurance: Mathematics & Economics.

Senel K., Pamukcu A. B., Yanik S., (2008), *An Evolutionary Approach to Asset Allocation in Defined Contribution Pension Schemes*, Brabazon A., O’Neill M., (ed.), **Natural Computing in Computational Finance**, (25–52), Berlin Heidelberg, Springer-Verlag.

Sherris M., (1993), “Portfolio Selection Models for Life Insurance and Pension Funds”, *Proceedings of Actuarial Approach for Financial Risks Colloquium*, 2 April 1993, 915-930.

Szmolyan N., (2008), “The Remaking of the DC Market”, Employee Benefit Research Institute 62nd Policy Forum, May 2008 (<http://www.ebri.org>).

Tuncer R., Senel K., (2004), “Optimal Investment Allocation Decision in Defined Contribution Pension Schemes with Time-Varying Risk”, 8th International Congress on Insurance: Mathematics and Economics.

Vigna E., Haberman S., (2001), “Optimal Investment Strategy for Defined Contribution Pension Schemes”, *Insurance: Mathematics and Economics*, (28), 233-262.

Willighagen E., (2005), R Based Genetic Algorithm, R package version 0.1.1.

Zhang X., Zhang K., (2009), “Using genetic algorithm to solve a new multi-period stochastic optimization model”, *Journal of Computational and Applied Mathematics*, (231), 114-123.